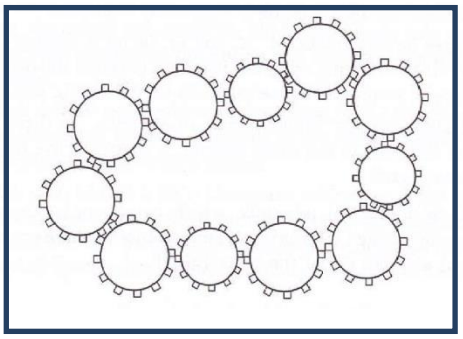
Parity Party



1. Gearing up for some maths

In the picture on the right, you see eleven gears in a chain.
a) Is it possible for all the gears to rotate at the same time?
b) What if there was an even number of gears?

2. Odds and Evens

In each of the following sums, find out whether the result is odd or even without actually computing it:

a. $1{,}256{,}827 + 7{,}571{,}269$
b. $999{,}999 - 888{,}888$
c. $10^{10} + 1$
d. 777×256
e. 131×99
f. $5 + 13 + 7 + 21 + 35$
g. $111 + 257 + 549 + 973$
h. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

Let's gather some general rules:

odd+odd=	odd−odd=
even+odd=	even−odd=
even+even =	even−even=
If I add an odd number of odd numbers together, the result is	odd × odd=
If I add an even number of odd numbers, the result is	even × any number=

Here is an example of an odd number of odd numbers: 3+5+11 (3 odd numbers)

3. An addition problem

a) If we add up the numbers 1 to 9 we get

1+2+3+4+5+6+7+8+9=45.

However we can change the plus signs to minus signs to get new numbers like

1+2+3+4+5-6-7-8-9=-15 or

-1+2-3+4-5+6-7+8-9=-5

Is it possible to switch some of the plus signs to minus signs so that the numbers add to zero?
That is, can you put plus and minus signs between the numbers (1 to 9) so the result is 0??
(Hint: Try for a smaller number first, for example 4 or 5.)
b) What if we have the same problem, but we take the numbers 1 to 100 instead?

4. A weighty problem

There is a box full of 1kg, 3kg, and 5kg weights.

a. Is it possible to take exactly 10 weights that together weigh 25kg?

b. I can make 25kg by using:

5 weights: 	$5 \times 5 = 25$
7 weights: 	$4 \times 5 + 1 \times 3 + 2 \times 1 = 25$

c. What other numbers of weights can you use to make 25 kg? List them all and explain.

5. A candy problem

You have lots of candies (or pieces of paper). You make two small piles, and keep the rest as reserve. *You can modify the piles **as many times as you like** by applying these rules **in any order:***

(i) You can take the same number of candies from each pile.

(ii) You can double the number of candies in <u>one</u> of the piles.

a) Play the game 3 times and see can you win.

a) Start with piles of 10 and 2 pieces. Can you win?

b) Can you find a way to finish the game with no candies, no matter which numbers of candies you started with in each pile?

c) If you start with piles of 10 and 3 pieces, but change the rules:

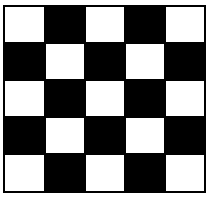
(i) You can take the same number of candies from each pile.

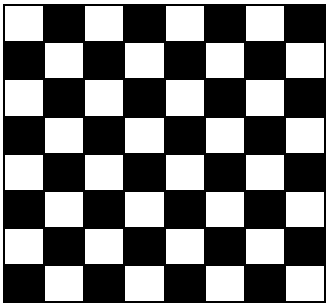
(iii) You can triple the number of candies in <u>one</u> of the piles.

Can you finish the game with no candies in each pile? Explain.

6. Domino problems

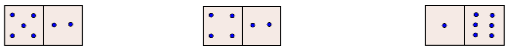
a) Is it possible to cover a 5×5 chessboard in dominoes? What about an 8×8 chessboard? Explain your answer in both cases.





A domino will cover two squares of the chessboard.

Dominoes:



b) If we take 1 corner square out of a 5×5 chessboard, is it possible to cover the chessboard with dominoes? Explain your answer.

c) If we take two opposite corner squares out of an 8×8 chessboard to make a "mutilated" chessboard, is it possible to cover it in dominoes? Explain your answer.

Parity Party II

1. Blackboard game

The numbers 1 to 2013 are written on a blackboard.

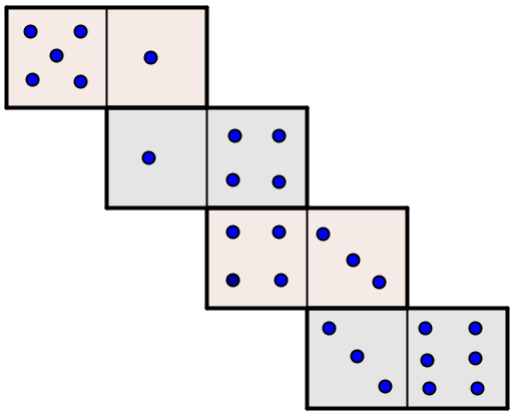
a) We can pick any two numbers from the blackboard, erase them and replace them with their sum. If we do this long enough, only one number will be left on the blackboard. What will that number be?

b) We start the game again, but this time we can pick any two numbers from the blackboard, erase them and replace them with their positive difference. If we do this long enough, is it possible for the only number on the blackboard to be zero?

2. More dominoes

A domino is a rectangle made up of two squares. Each square has 0 to 6 dots on it. A set contains one of each kind of domino.

a. How many dominoes are there in a set?
b. All the dominoes in a set are lined up in a chain (so that the number of dots on the ends of side-by-side dominoes match). If the square at one end has 5 dots on it, how many dots does the square on the other end have?
c. If we take out all the dominoes that have a square with no dot from a set, can we make a chain with the left over dominoes?



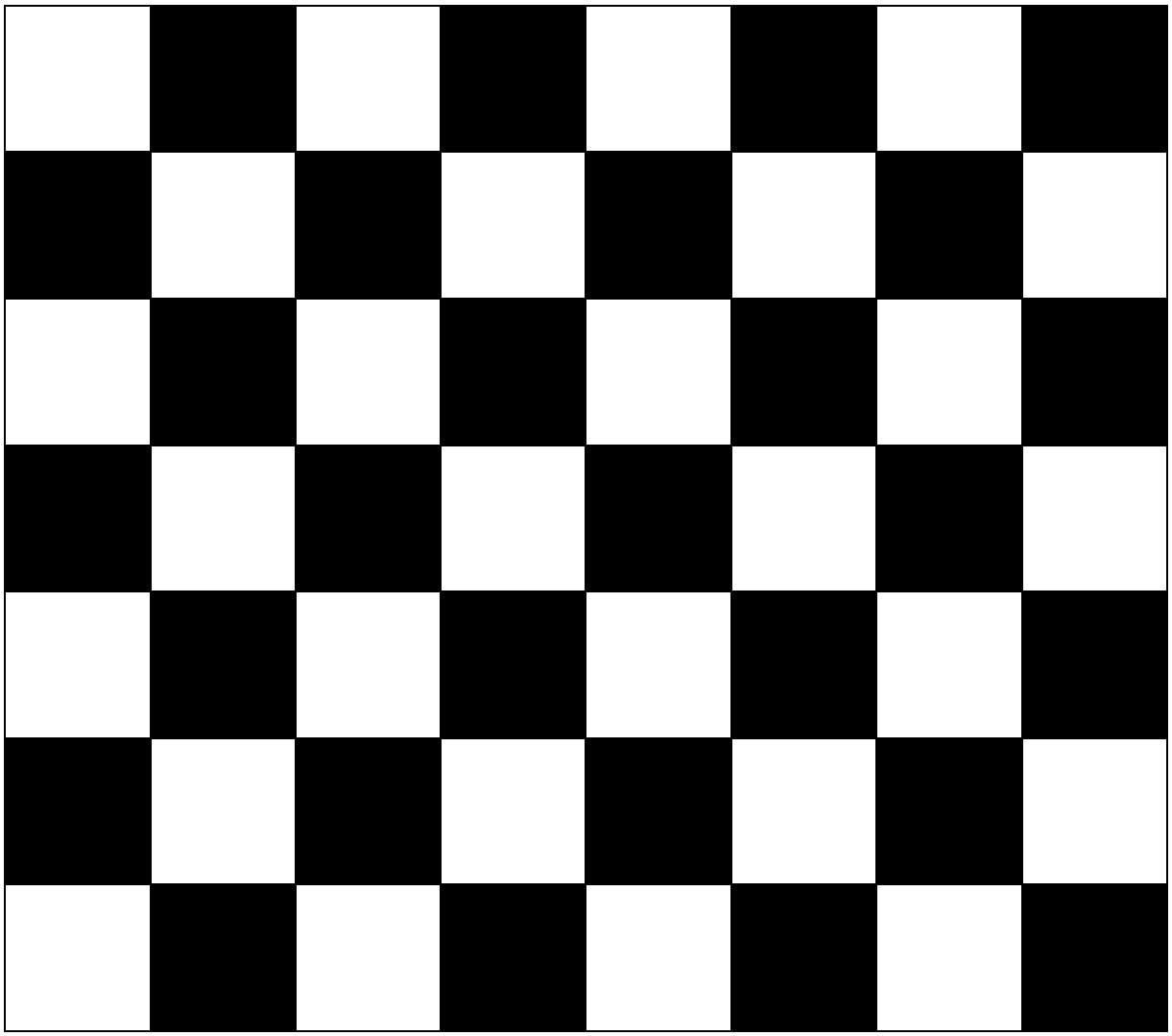
3. Chessboard problems

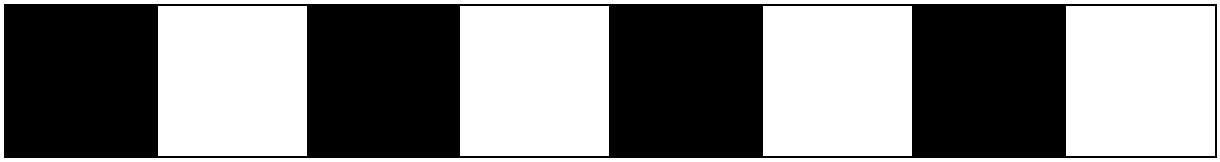
a) On a chessboard, a knight makes an “L-shaped” move, made up of moving 2 squares in 1 direction, and 2 squares in another direction. If a knight starts on one corner square of the board, what is the fewest number of moves it takes for the knight to get to the opposite corner of the board?

b) Is it possible for the knight to get from one corner to the other while landing on every other square on the board exactly once?

c) Is it possible for the knight to be on any square in the board (after starting from the corner) after exactly 6 moves?

d) Is there any number, such that starting from a corner square, the knight can get to any square on the board it likes in exactly that many moves?





Zero-One Arithmetic

Remember the even/odd rules:

even+even =even
even+odd=odd
odd+odd= even

Sometimes people are too lazy to write “odd” and “even” and so write “1” for all odd numbers and “0” for all even numbers. This is called mod 2 arithmetic, and in one respect it is different from the usual sum rules:

Mod 2 arithmetic:

0+0=0
1+0=1
1+1=0

It’s like turning a light switch: if you turn it once, light is on.  But if you turn it twice, light is off.

1. Light switch game

Four Christmas light bulbs are arranged in a 2×2 grid. The switches for this grid are so connected that whenever you turn the switch for one of the light bulbs, this also affects the light bulbs immediately up or down, right or left from it. I’d like to know all the possible patterns of light I can get.

a) Examples:

Unlit grid:

0	0
0	0

In the unlit grid, turn the left upper corner light switch:

1	1
1	0

In the unlit grid, turn the right upper corner light switch:

1	1
0	1

In the unlit grid, turn the left lower corner light switch:

In the unlit grid, turn the right lower corner light switch:

Now let’s turn the switches in succession:

Left Up + Right Up:

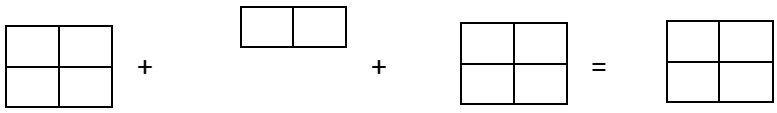
1	1
1	0

+

1	1
0	1

=

Left Up + Right Up + Right Low:



b) How many patterns are there in total? If you start from the unlit grid, how many patterns can you get by turning switches?

c) If you start from

1	0
0	0

how can you get to

0	0
1	0

?

d) If you start from

1	0
0	0

how can you get to

0	0
0	1

?

2. Coin game

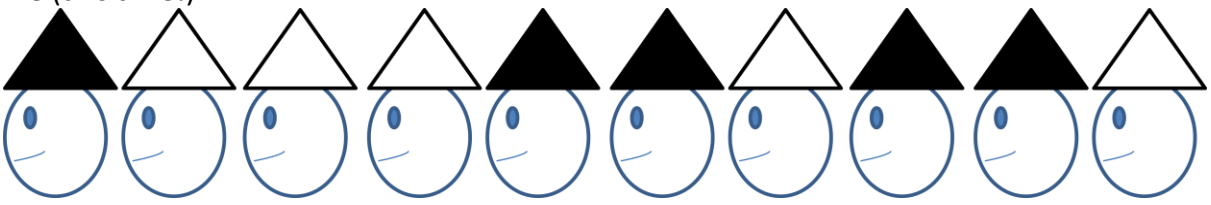
One face shows the denomination (1 cent, 2 cent, 5 cent, 10 cent). Let's call this face "Cent".
The other face has a 12 star circular border. Let's call this face "Star".

Arrange 16 coins on a table, so that they all have the same face up. One player is turned away from the table, eyes closed. Another player flips any of the coins on the board any number of times, each time saying "Tap", and at the end covers one of the coins with the hand. The first player turns around and has to guess which face of the covered coin is up.

Is there a strategy for guessing all the time?

3. Hat game

Being an extremely cruel and vicious maths teacher, I have decided to take ten of the students and stand them in a line one behind the other. On each of their heads I will place a hat, either black or white, I don't care how many of each colour. Each person will see the colours of all the hats ahead of them, but be unable to see those behind them or their own. The kids starting from the back must tell what colour hat they are wearing, if they pick the wrong colour unfortunately they will die. However if they pick the right colour I will let them live (this time!).



Before we start this whole process, the students are allowed to discuss and to come up with the best strategy in order to save as many as possible…and just so you know you can definitely save at least nine of the kids!!

3. Chocolate unwrapping game

http://funschool.kaboose.com/arcade/games/game_chocolate_biz.html

a) Can you describe the game in 0-1 algebra? How would you represent each 1 move?
b) If you start with a completely wrapped chocolate and then click on each of its squares exactly once, what pattern do you get at the end?

c) If you start with a completely wrapped chocolate and then click on the corner squares and the 4 interior squares, what pattern do you get at the end?
d) How can you win the game in the least possible number of moves?