1. The numbers 1 to 2019 are written on a blackboard.
   a) We can pick any two numbers from the blackboard, erase them and replace them with their sum. If we do this long enough, only one number will be left on the blackboard. What will that number be?
   b) We start the game again, but this time we can pick any two numbers from the blackboard, erase them and replace the answer of the bigger number minus the smaller number. If we do this long enough, is it possible for the only number on the blackboard to be zero?

2. An even number is a number that is divisable by 2, with remainder 0. An odd number is a number with a remainder of 1 when you divide by 2. Let's use 1 to represent odd numbers, and 0 to represent even numbers. Then we have the following rule:

even + even = even	0+0 =0	even × even =	$0 \times 0 =$
even + odd = odd	0+1=1	even × odd =	
odd + odd = even	1+1 =0	odd × odd =	

   a) Can you fill in the box above?
   b) If you add together an even number of odd numbers (ex. 7+3+5+21), you get an even number. Can you explain this using 1 and 0?
   c) What happens when you add an odd number of odd numbers?
   d) Can you draw the first table, but replace + with −?

3. You have 2 small pieces of paper, one side of each piece has a black mark on it. You play the following two player game, starting with all the papers with black mark side facing up.
   i. Player A turns away from the table.
   ii. Player B turns over some pieces of paper, and says tap at every turning. (She/he can turn the same piece of paper as many times at she/he likes.)
   iii. Player B covers one piece of paper with their hand and Player A turns to look at the table and has to guess whether the covered paper has the black mark facing up.

   a) Play the game four times (swapping each time who is Player A and who is Player B.) Is there a strategy so that Player A always wins?

   b) Now play the game with 16 pieces of paper instead of 2 pieces. Is there a strategy so that Player A always wins?

4. I play the following game with 10 students. They must stand in a line, all facing the same direction. Each student can see the students in front of him, but has no information about the students behind him. I put a cap on each student, which is either black or white. The students can't see their own cap.

   The student at the back must guess out loud what colour hat he is wearing, then the student in front of him and so on, until all the students have guessed. Apart from hearing each others guesses, they are not allowed to communicate in any way. Each student can only say ”black” or ”white”.

   They win if at least 9 of the students guess correctly. They can talk to each other before they play to make a strategy. What should the strategy be?

3. Chessboard problems

a) On a chessboard, a knight makes an “L-shaped” move, made up of moving 2 squares in 1 direction, and 1 square in another direction (not counting the starting square). If a knight starts on one corner square of the board, what is the fewest number of moves it takes for the knight to get to the opposite corner of the board?

b) Is it possible for the knight to get from one white corner to the opposite one while landing on every other square on the board exactly once?

c) Is it possible for the knight to be on any square on the board (after starting from the corner) after exactly 6 moves?

d) Is there any number, such that starting from a corner square, the knight can get to any square on the board it likes in exactly that many moves?

3 example moves are drawn. Start at the first x, then move 2 right, 1 down to get to the second x. Then 2 right, 1 down to 3rd x. Then 2 down, 1 left to get to 4th x.