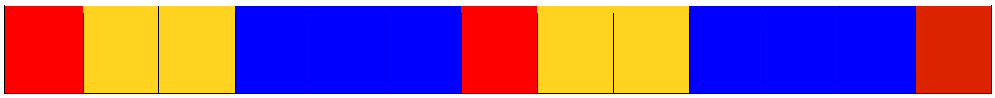
Cool Counting

1. A colour pattern

Eoin is painting a border of 2,000 tiles in the following pattern:



And so on. What colour will be the last tile? How many yellow tiles are there in total?

2. How many numbers?

a) How many numbers are there in the list

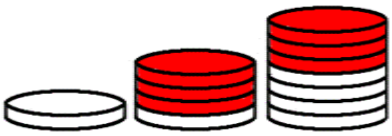
$$37, 38, 39, \ldots, 100?$$

b) How many numbers are there between each of the following pairs of numbers, if we count the beginning and the end too?

$i)\ 1{,}000{,}000\ and\ 2{,}000{,}000 \qquad ii) -50\ and -25 \qquad iii) -102\ \ and\ \ 27$

3. What number comes next?

a) What is the 23nd multiple of 5 in the list $40, 45, 50, 55, \ldots$?
b) What is the 21st number in the list 23, 25, 27, 29, … ?
c) What is the 26th number in the list 25, 28, 31, 34, 37, …?
d) What is the 301st number in the list 2013, 2008, 2003, 1998, …?
e) What is the 50th number in the list 1, 3, 6, 10, 15, 21, …?



4. More lists of numbers

a) How many even numbers are there in the list $22, 23, 24, \ldots, 143, 144$?
b) How many odd numbers are there in the list $22, 23, 24 \ \ldots, 143, 144$?
c) How many numbers in the list $22, 23, 24 \ \ldots, 143, 144$ are divisible by 5?
d) How many numbers in the list above are divisible by neither 2 nor 5?

5. More devious lists of numbers

a) How many 3 digit numbers are divisible by 7?
b) How many 3 digit numbers are perfect squares
(meaning that they are the square of another whole number)?
c) How many numbers are there in the list 27, 38, 49, 60..., 2018?
d) How many numbers are there in the list
1, 2, 5, 10, 17, 26, 37, 50, ..., 485?

Extension Questions

1. How many numbers are there such that if you multiply the number by 3 or if you divide the number by 3, you will have a 3-digit whole number?

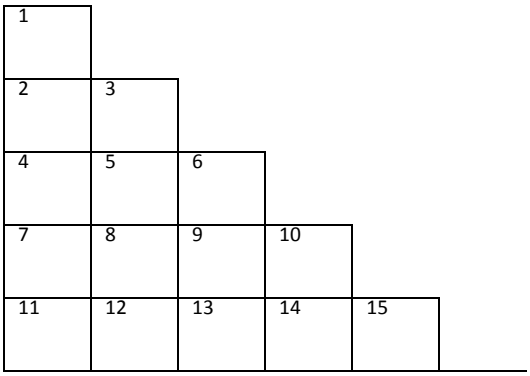
2. How many times does the digit 5 occur in the numbers 1 to 999?

3. How many of the numbers from 1 to 999 have at least one digit of 5, e.g. 315 and 524?

4. Consider the number $123456 \ldots 998999$, which is formed by writing the numbers 1,2,3,4,…,999 in order. What is the 2012th digit from the left?

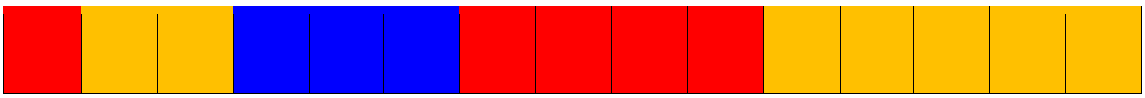
5. What is the most amount of Sundays you can have in 1 month? What about in 1 year?

6. June 17th 2012 was a Sunday. What was the day of the week on 17th June, 1998?
   What will the date be 1 million minutes after noon 1st January 2012?



7. The positive integers are arranged in the pattern indicated in the diagram. What number will be found in the square for the 61st (horizontal) row and 23rd (vertical) column?

8. Eoin is painting a border of 5,050 tiles in the following pattern:



And so on. What colour will be the last tile?
How many blue tiles are there in total?
How many yellow tiles are there in total?
How many red tiles are there in total?